ESTIMATING THE MODIFIED ALLAN VARIANCE

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A paper at the 1992 FCS showed how to express the modified Allan variance (mvar) in terms of the third difference of the cumulative sum of time residuals. Although this reformulated definition was presented merely as a computational trick for simplifying the calculation of mvar estimates, it has since turned out to be a powerful theoretical tool for deriving the statistical quality of those estimates in terms of their equivalent degrees of freedom (edf), defined for an estimator V by edf $V = 2(EV)^2/(\mathrm{var}\ V)$. Confidence intervals for mvar can then be constructed from levels of the appropriate χ^2 distribution.

Using processes with $\sin^\beta(\pi f \tau_0)$ spectra as discrete-time power-law phase noise models, I obtain the following results. (1) Expressions for mvar estimator edf, more tractable than previous expressions. As a paper at this year's EFTF shows, these expressions agree numerically with the previous expressions and with extensive simulations. (2) An assessment of the dependence of the edf of an mvar estimator on its stride, defined as the time interval by which the summands of the estimator are shifted. Numerical computations show empirically that any stride between the sample period τ_0 and 1/4th the averaging time, subject to a divisibility condition, gives essentially the same estimator edf. (3) A simple approximation formula for edf, with coefficients drawn from a brief lookup table. Its accuracy has been verified empirically. Most users will not need the exact edf formulas. (4) A theorem allowing conservative values of estimator edf to be obtained in the presence of compound phase noise spectra, i.e., linear combinations of pure power laws with unknown coefficients. Because the edf of an mvar estimator usually varies less with power-law exponent than the edf of the corresponding avar (conventional Allan variance) estimator does, the usefulness of this result for mvar is enhanced.

The 1992 paper showed that mvar estimates are almost as easy to calculate as avar estimates. The current results show that the *confidence* of mvar estimates is actually *easier* to approximate than the confidence of avar estimates, and is more robust against spectral uncertainties.

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